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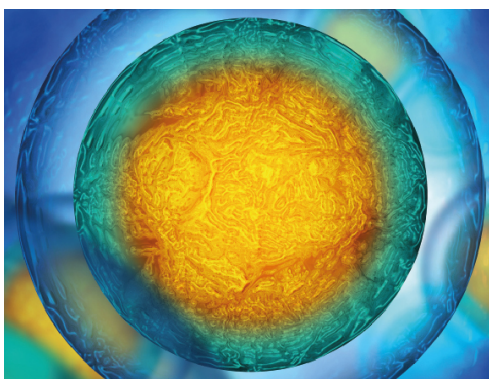
A general locomotion control framework for multi-legged locomotors

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A general locomotion control framework for multi-legged locomotors

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Abstract

Serially connected robots are promising candidates for performing tasks in confined spaces such as search and rescue in large-scale disasters. Such robots are typically limbless, and we hypothesize that the addition of limbs could improve mobility. However, a challenge in designing and controlling such devices lies in the coordination of high-dimensional redundant modules in a way that improves mobility. Here we develop a general framework to discover templates to control serially connected multi-legged robots. Specifically, we combine two approaches to build a general shape control scheme which can provide baseline patterns of self-deformation ('gaits') for effective locomotion in diverse robot morphologies. First, we take inspiration from a dimensionality reduction and a biological gait classification scheme to generate cyclic patterns of body deformation and foot lifting/lowering, which facilitate the generation of arbitrary substrate contact patterns. Second, we extend geometric mechanics, which was originally introduced to study swimming at low Reynolds numbers, to frictional environments, allowing the identification of optimal body–leg coordination in this common terradynamic regime. Our scheme allows the development of effective gaits on flat terrain with diverse numbers of limbs (4, 6, 16, and even 0 limbs) and backbone actuation. By properly coordinating body undulation and leg placement, our framework combines the advantages of both limbless robots (modularity and narrow profile) and legged robots (mobility). Our framework can provide general control schemes for rapid deployment of general multi-legged robots, paving the way toward machines that can traverse complex environments. In addition, we show that our framework can also offer insights into body–leg coordination in living systems, such as salamanders and centipedes, from a biomechanical perspective.

1. Introduction

Robot performance characteristics can depend on the number of limbs. Quadrupeds are known for their agility [1], hexapods and myriapods for their stability

[2, 3], and limbless robots for their ability to fit into confined spaces [4]. But robots with increasing complexity and numbers of degrees-of-freedom (DoF) present challenges regarding motion coordination, which if not addressed, may render them unusable.

Furthermore, the diversity of shapes and forms makes it challenging to transfer control insights gained from one platform onto another. We are left with limited intuition and physical understanding of how to coordinate the many DoF in diverse and complex robots to generate effective locomotion.

To address the growing need to control robots with different shapes, modular robot control¹¹ strikes a balance between encompassing a variety of shapes while still being able to precisely control them [6, 7]. Modular robot control has been successfully used in serially connected limbless robots where a single control principle can be applied in robots of different sizes [8]. In contrast, the study of modular control in general multi-legged robots has been limited. The challenge in serially connected multi-legged robots lies not only in designing the stepping patterns of legs, but also in the coordination between the body and legs. For example, in robots that combine limbs and body undulation, if stepping patterns and body undulations are not properly coordinated, limbs can interfere with each other, resulting in reduced locomotor performance, instability, or even failure [9, 10].

We would like to develop control schemes to generate effective periodic ‘self-deformation patterns’¹² for the general class of serially connected legged and limbless robots. Over recent decades, many techniques (e.g., gait generation [11, 12], central pattern generators [13, 14], nearest limb synchronization [15], and learning methods [1, 16]) have been developed, each of which can control some specific robot type [3, 13, 17–19]. In this paper, we take inspiration from gaits of living systems: organisms with diverse numbers of appendages and body plans exhibit effective locomotion on almost all terrestrial environments [19–21] by making/breaking ground contact with limbs (e.g., salamanders) and bodies (e.g., sidewinders) in conjunction with waves of undulation.

One method used over the last century to understand legged locomotion is a gait classification scheme called ‘Hildebrand diagrams’. In 1965, Hildebrand [22] developed schemes to study symmetric gaits¹³ observed in quadrupedal animals (e.g., horses). These gaits have two key variables: *duty factor*, the fraction of a period that each leg is on the ground over a full gait cycle, and *lateral phase lag*, the fraction of a period that the hind leg leads the foreleg on the same side. Both key variables are modulated in response to speed changes in biological systems [23–25]. Using these gait principles as a reference, a multitude of algorithms have been developed for quadrupedal robot

locomotion, or to explain why living quadrupeds choose certain gaits [26–31]. But thus far, these gait principles have not been applied to robots with more than four appendages. For multi-legged robots, there is no systematic gait description framework that allows us to modulate the balance between locomotion metrics such as speed and stability.

In animals and increasingly in robots, appendages that make direct contact with substrates are not the sole contributor to locomotion. Undulatory body motions play an important role in generating propulsive forces in many systems [20, 32–34]. For undulatory locomotors, the geometric mechanics (GM) community [35–42] has developed a gait design framework to prescribe self-deformations of systems immersed in continuous media, such as three-link robots, lizards, and snakes [18, 39, 43]; and in discontinuous settings, including sidewinders [44, 45]. While mathematically elegant, GM has limitations. In particular, it is not directly applicable to systems with a large number of appendages. Furthermore, despite recent efforts [43, 46], application of GM in frictional environments (e.g., rate-independent Coulomb dry friction) has not been systematically studied. Therefore, we must develop dimensionality reduction and physical modeling methods before we can use GM to design gaits for serially connected multi-legged robots.

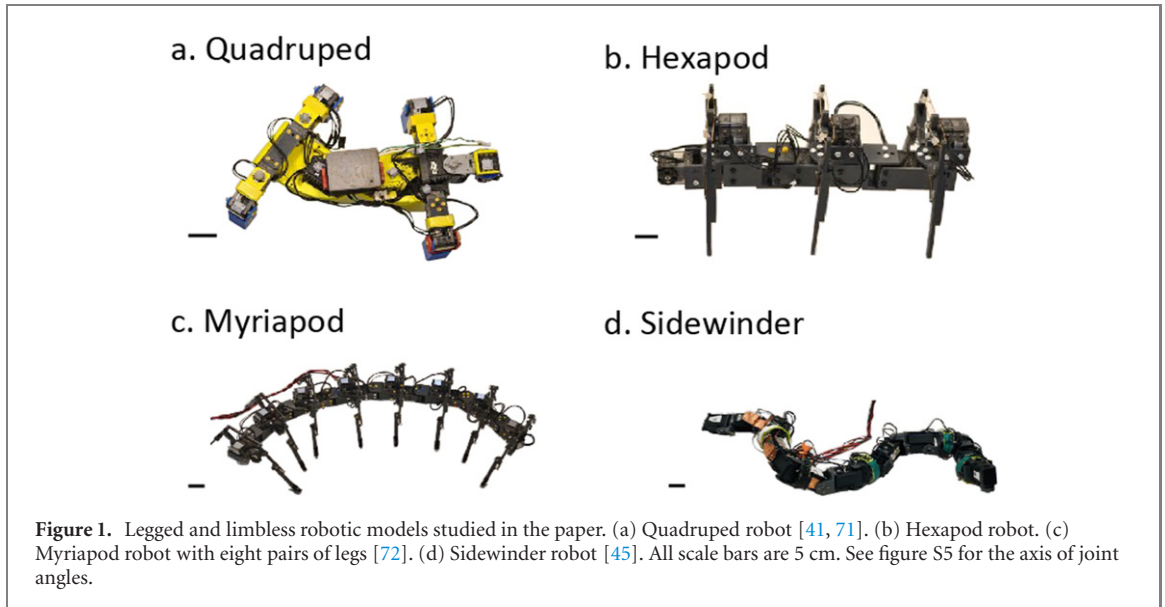
In this paper, we integrate dimensionality reduction techniques with tools from GM to develop locomotion control schemes for serially connected robots. We first extend the Hildebrand gait classification scheme to prescribe a wide range of contact patterns (the sequence of making/breaking contacts with environments) using the classical Hildebrand parameters (duty factor and lateral phase lag). We use the extended Hildebrand scheme to reduce dimensionality and prescribe body undulation as a traveling wave. In doing so, we can apply GM to coordinate the lateral body undulation and limb contact patterns. We evaluate gait performance based on speed and static stability, and investigate the relationship between these metrics and the Hildebrand parameters. We demonstrate our motion control framework on robots with four (quadrupedal), six (hexapod), 16 (myriapod-like), and even zero (snake-like) limbs (figure 1). Our analysis reveals empirical rules to balance the trade-off between speed and static stability, and the potential benefit of body undulation in multi-legged robot locomotion.

Moreover, by properly coordinating lateral body undulation and leg movement, our framework provides additional insights into both legged and limbless robots. Specifically, our framework facilitates centralized control of serially connected multi-legged robots by introducing waves in both limb contact and lateral body undulation. With properly coordinated lifting and landing body segments, our framework can also improve the mobility of limbless robots by

¹¹ A control framework where the policy structure is conditioned on the hardware arrangement, and uses just one training process to create a policy that controls a wide variety of designs [5].

¹² We consider self-deformation patterns to be the relative movement of body and limb elements.

¹³ In symmetric gaits, the contralateral (left and right pair) of legs are 180° out of phase.



giving insights into coordination and trade-offs of stability and speed in serially connected multi-legged robots. In this way, our framework offers the potential to modulate gaits for different tasks by switching between fast gaits and stable gaits. Further, we show that our scheme can generate control hypotheses for diverse living systems including salamanders and centipedes, thereby offering new insights into the functional role of body–leg coordination from a biomechanical and robophysical perspective.

2. Hildebrand gait prescription

2.1. Related work

In the Hildebrand gait formulation [22], symmetric quadrupedal gaits are categorized by two parameters: 1) duty factor that represents the fraction over a gait period that each leg is on the ground, and 2) lateral phase lag represents the fraction over a gait period that the hind leg leads the foreleg on the same side. There are three major assumptions in the Hildebrand symmetric gait family: (1) the duty factor of each leg is the same, (2) the pairs of contralateral legs are 180° out of phase, and (3) the lateral phase lag is the same for left and right legs.

We use a binary variable c to represent the contact state of a leg, where $c = 1$ represents the stance phase and $c = 0$ represents the swing phase. The contact pattern of symmetric quadrupedal gaits can be written as

$$\begin{aligned}
 c_{\text{FL}}(\phi_c) &= \begin{cases} 1, & \text{if } \text{mod}(\phi_c, 2\pi) < 2\pi D \\ 0, & \text{otherwise} \end{cases} \\
 c_{\text{FR}}(\phi_c) &= c_{\text{FL}}(\phi_c + \pi) \\
 c_{\text{HL}}(\phi_c) &= c_{\text{FL}}(\phi_c + 2\pi\Phi_{\text{lat}}) \\
 c_{\text{HR}}(\phi_c) &= c_{\text{FL}}(\phi_c + 2\pi\Phi_{\text{lat}} + \pi), \quad (1)
 \end{aligned}$$

where Φ_{lat} denotes the lateral phase lag, D the duty factor, $c_{\text{FL}}(\phi_c)$, $c_{\text{FR}}(\phi_c)$, $c_{\text{HL}}(\phi_c)$, and $c_{\text{HR}}(\phi_c)$ the contact state of the fore-right (FR), fore-left (FL), hind-left (HL), and hind-right (HR) limbs at the gait phase ϕ_c (defined in radians with a period of 2π), respectively. Note that in contrast to ϕ_c , the lateral phase lag Φ_{lat} represents the fraction of a cycle, so Φ_{lat} has a period of 1. Many common quadrupedal gaits can be described using the Hildebrand formula. For example, the lateral sequence (LS) walking gait (figure 2) can be described by $D = 0.75$, $\Phi_{\text{lat}} = 0.25$. Plotting a diagram of the stance/swing phases of the feet from just these two parameters shows that in this gait, each leg is lifted for a quarter of a cycle, only one leg is lifted at any given instant, and the leg lifting sequence follows FR, HR, FL, and HR. The trot gait (figure 2) can be described by $D = 0.5$, $\Phi_{\text{lat}} = 0.5$, where the FR and HL are coupled in phase, as are the FL and HR pair. Another quadrupedal gait, the pace gait (figure 2), can be described by $D = 0.5$, $\Phi_{\text{lat}} = 0$, where the FR and HR are coupled in phase, as are the FL and HL pair. Note that asymmetric quadrupedal gaits, such as bounding and galloping, exist but cannot be prescribed by the same gait classification methods [47].

2.2. Prescription of contact patterns for arbitrary robots

The first two assumptions of the Hildebrand symmetric gait family can hold in general for nonquadrupedal systems with discrete contacts. To expand the third assumption to a broader range of locomotors, we can generalize the definition of the lateral phase lag to be the phase lag between two consecutive legs (instead of only the fore and hind legs) on the same side. Then, the contact function of a multi-legged system can be written as:

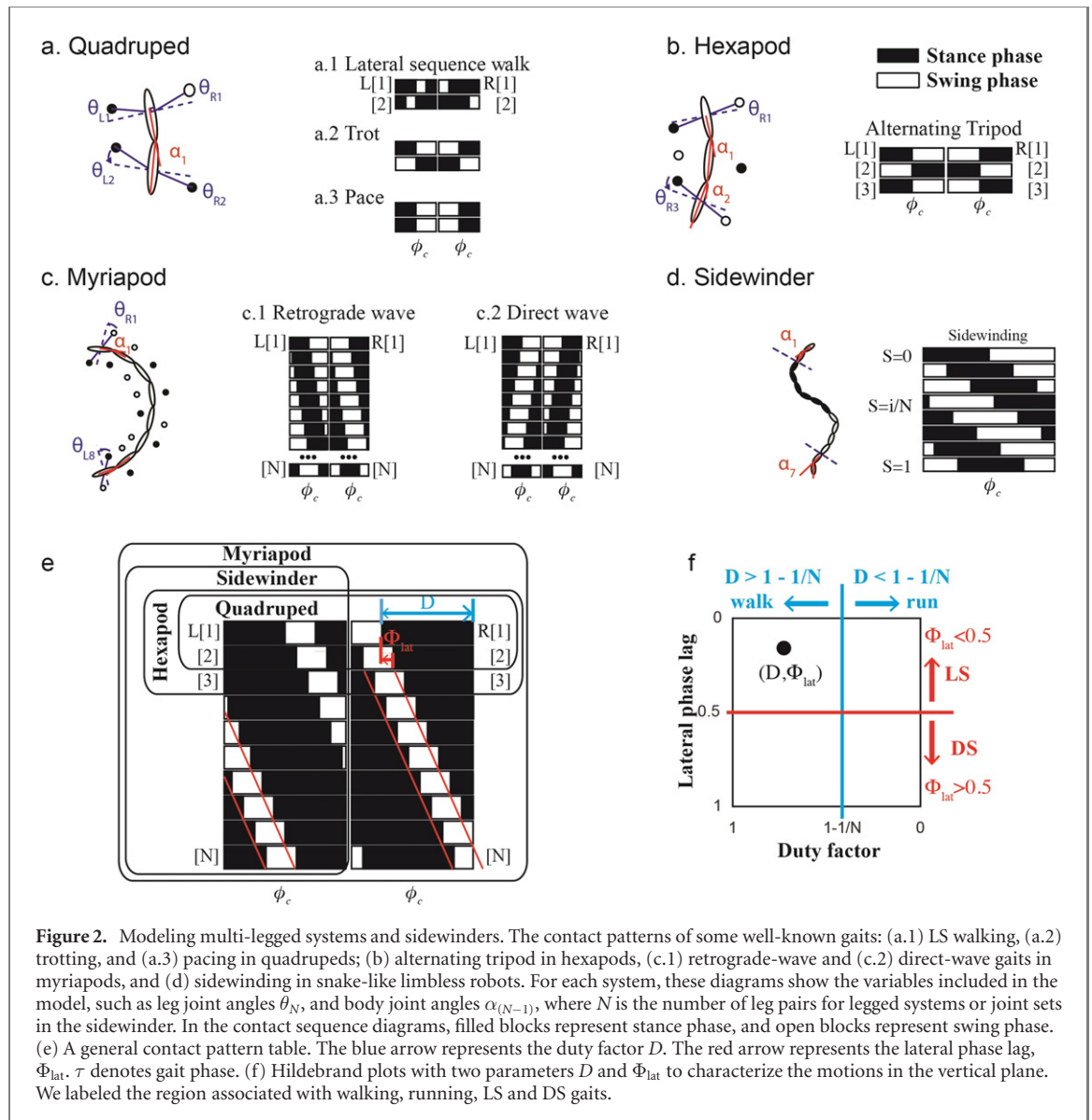


Figure 2. Modeling multi-legged systems and sidewinders. The contact patterns of some well-known gaits: (a.1) LS walking, (a.2) trotting, and (a.3) pacing in quadrupeds; (b) alternating tripod in hexapods, (c.1) retrograde-wave and (c.2) direct-wave gaits in myriapods, and (d) sidewinding in snake-like limbless robots. For each system, these diagrams show the variables included in the model, such as leg joint angles θ_N , and body joint angles $\alpha_{(N-1)}$, where N is the number of leg pairs for legged systems or joint sets in the sidewinder. In the contact sequence diagrams, filled blocks represent stance phase, and open blocks represent swing phase. (e) A general contact pattern table. The blue arrow represents the duty factor D . The red arrow represents the lateral phase lag, Φ_{lat} . τ denotes gait phase. (f) Hildebrand plots with two parameters D and Φ_{lat} to characterize the motions in the vertical plane. We labeled the region associated with walking, running, LS and DS gaits.

$$c_l(\phi_c, 1) = \begin{cases} 1, & \text{if } \text{mod}(\phi_c, 2\pi) < 2\pi D \\ 0, & \text{otherwise} \end{cases}$$

$$c_l(\phi_c, i) = c_l(\phi_c + 2\pi(i-1)\Phi_{lat}, 1)$$

$$c_r(\phi_c, i) = c_l(\phi_c + \pi, i), \quad (2)$$

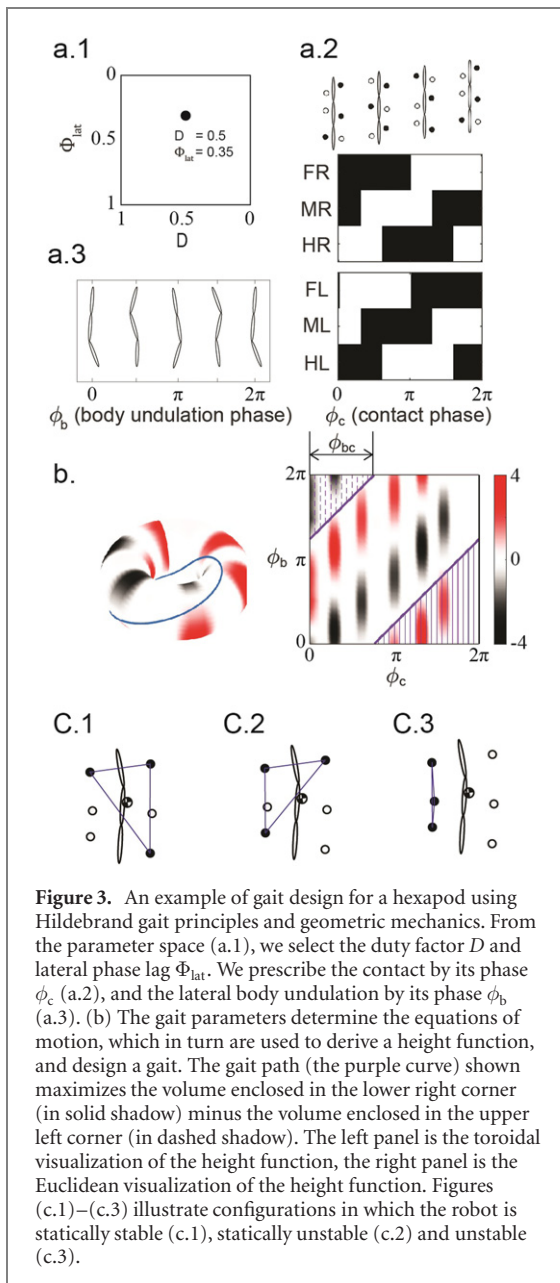
where $c_l(\phi_c, i)$ (and $c_r(\phi_c, i)$) denotes the contact state of the i th leg on the left (and the right) at gait phase ϕ_c , $i \in \{1, \dots, N\}$ for $2N$ -legged systems.

Many common multi-legged gaits can also be described by this extended Hildebrand formulation. For example, many hexapod robots and animals use the alternating tripod gait (figure 2), which couples FL, middle-right (MR), and HL in phase, and couples the FR, ML, and HR similarly. The alternating tripod gait for a hexapod ($N = 3$) can be described by $D = 0.5$ and $\Phi_{lat} = 0.5$.

Myriapod gaits can be classified into direct waves and retrograde waves of limb contact [48] (figure 2). Typically, for gaits with $\Phi_{lat} < 0.5$, the phase of the

hind leg is ahead of the phase of its immediate fore leg. In other words, the legs move in a wave propagating from tail to head, which we call a diagonal sequence (DS) gait, and which corresponds to direct waves in myriapods. On the other hand, when $\Phi_{lat} > 0.5$, the phase of the hind leg is behind the phase of its immediate fore leg. Therefore, the leg wave propagates from head to tail, which we call a LS gait, and which corresponds to retrograde waves in myriapods. Interestingly, on level ground, animals with four legs (e.g., lizards and salamanders) more commonly use LS gaits [22, 23, 49, 50], and animals with more legs (e.g., centipedes) use both DS and LS gaits [34, 51]. As we will discuss later, we hypothesize that this difference in gait choice is a result of a balance between speed and stability.

Our proposed gait formulation can also include systems without legs, e.g., sidewinding limbless robots. The seemingly complex mode of limbless locomotion, sidewinding, can be prescribed as the superposition of two waves: lateral and vertical body



waves [40]. Similar to legged systems, sidewinders can regulate their contacts by modulating the vertical traveling wave [40]. The typical contact pattern of a sidewinder is shown in figure 2. Note that the contact pattern during sidewinding locomotion is the same as one side (either left or right) of the contact pattern of a legged system. As such, we prescribe the contact state of the i th link of the sidewinding system as $c(\phi_c, i) = a_1(\phi_c, i)$, where $a_1(\phi_c, i)$ is defined in equation (2).

2.3. Prescription of leg shoulder movement

Legs generate self-propulsion by protracting during the stance phase to make contact with the environment, and retracting during the swing phase to break contact. That is, the leg moves from the anterior to the posterior end during the stance phase and moves from the posterior to anterior end during the swing phase.

With this in mind, we use a piece-wise sinusoidal function to prescribe the anterior/posterior excursion angles (θ , figure 2) for a given contact phase (ϕ_c) defined earlier,

$$\theta_l(\phi_c, 1) = \begin{cases} A_\theta \cos\left(\frac{\phi_c}{2D}\right), & \text{if } \text{mod}(\phi_c, 2\pi) < 2\pi D \\ -A_\theta \cos\left(\frac{\phi_c - 2\pi D}{2(1-D)}\right), & \text{otherwise,} \end{cases}$$

$$\theta_l(\phi_c, i) = \theta_l(\phi_c + 2\pi(i-1)\Phi_{\text{lat}}, 1)$$

$$\theta_r(\phi_c, i) = \theta_l(\phi_c + \pi, i), \quad (3)$$

where A_θ is the shoulder angle amplitude, $\theta_l(\phi_c, i)$ and $\theta_r(\phi_c, i)$ denote the leg shoulder angle of i th left and right leg at contact phase ϕ_c , respectively. Note that the shoulder angle is maximum ($\theta = A_\theta$) at the transition from swing to stance phase, and is minimum ($\theta = -A_\theta$) at the transition from stance to swing phase. Figure 3 shows an example of a hexapod gait under this equation.

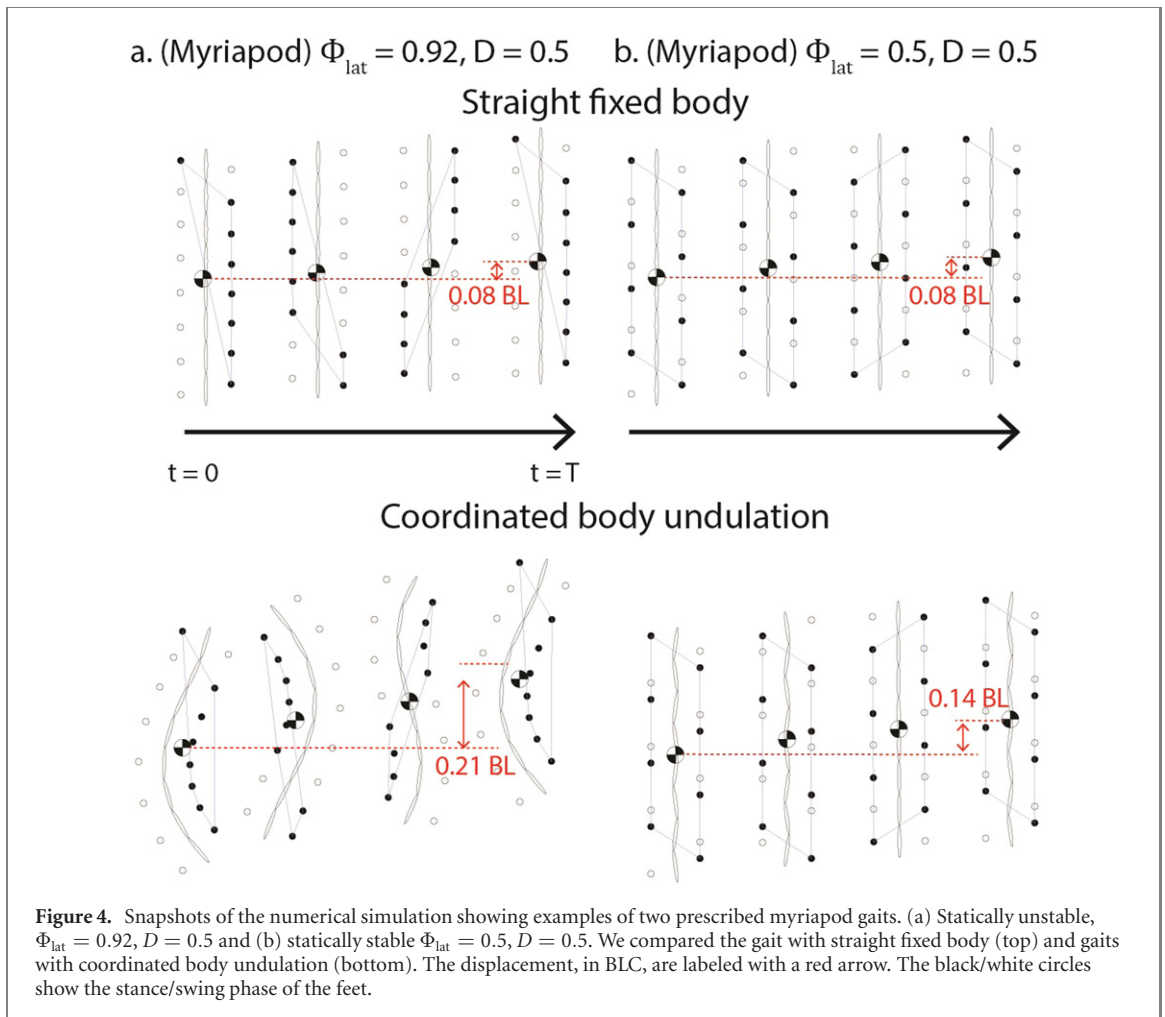
2.4. Numerical prediction on speed and stability

We numerically calculated the speed of various gaits over a range of duty factors and lateral phase lags for quadrupedal, hexapod, myriapod, and sidewinder systems ([39] and materials and methods). Figures 3 and 4 graphically depict the process, and the materials and methods section provides details. To explicitly show the effect of limb-substrate contact on speed, we fixed the swing angle A_θ when comparing the displacements of different gait parameters. Note that in this section, there is no body undulation in any of the gaits.

The numerical predictions of body speed, measured in units of body length per cycle (BLC), are plotted in figure 5 (middle column). We observe that modulating the lateral phase lag does not significantly affect body speed. This observation becomes more apparent for systems with more legs. In the myriapod system, the step displacement¹⁴, characterized by displacement (normalized by body length) traveled per cycle, is almost independent of the lateral phase lag and is uniquely determined by the duty factor.

In addition to measuring body speed, we utilize other metrics to quantify gait stability. For instance, the contact pattern of quasi-static gaits (e.g., quadrupedal walking gaits) need significantly fewer sensors and less feedback control effort to be stably implemented on robots than the contact pattern of dynamically stable gaits (e.g., bouncing gaits) [26]. In this paper, we separate robots' configurations into three groups: (1) statically stable, (2) statically unstable, and (3) unstable. In the statically stable configurations, the center of mass is bounded

¹⁴ The absolute speed v is related to the step displacement d by $v = fd$, where f is the step frequency.



within the supporting polygon (figure 3(c.1)). In the statically unstable configurations, often produced by unstable diagonal-couplet gaits [49], the center of mass is outside the supporting polygon but there is at least one leg in stance phase on the left and the right side (figure 3(c.2)). Despite not being statically stable, the statically unstable configurations can be made dynamically stable when the speed increases [23] or when combined with a low-level controller [17, 29, 30]. In the unstable configurations, also known as unstable lateral-couplet gaits [49], either the left or the right side of the legs are all in swing phase (figure 3(c.3)), which makes such gaits more difficult to stabilize¹⁵. We define a static stability metric as the fraction of the gait cycle spent in statically stable configurations. Note that this measure only applies to the gaits with statically stable and statically unstable configurations; the appearance of unstable configurations will contradict our assumptions. Therefore, we define the measure of static stability to be 0 if there exists unstable configurations in the gait.

We numerically calculated the static stability for the quadrupedal, hexapod, and myriapod systems in

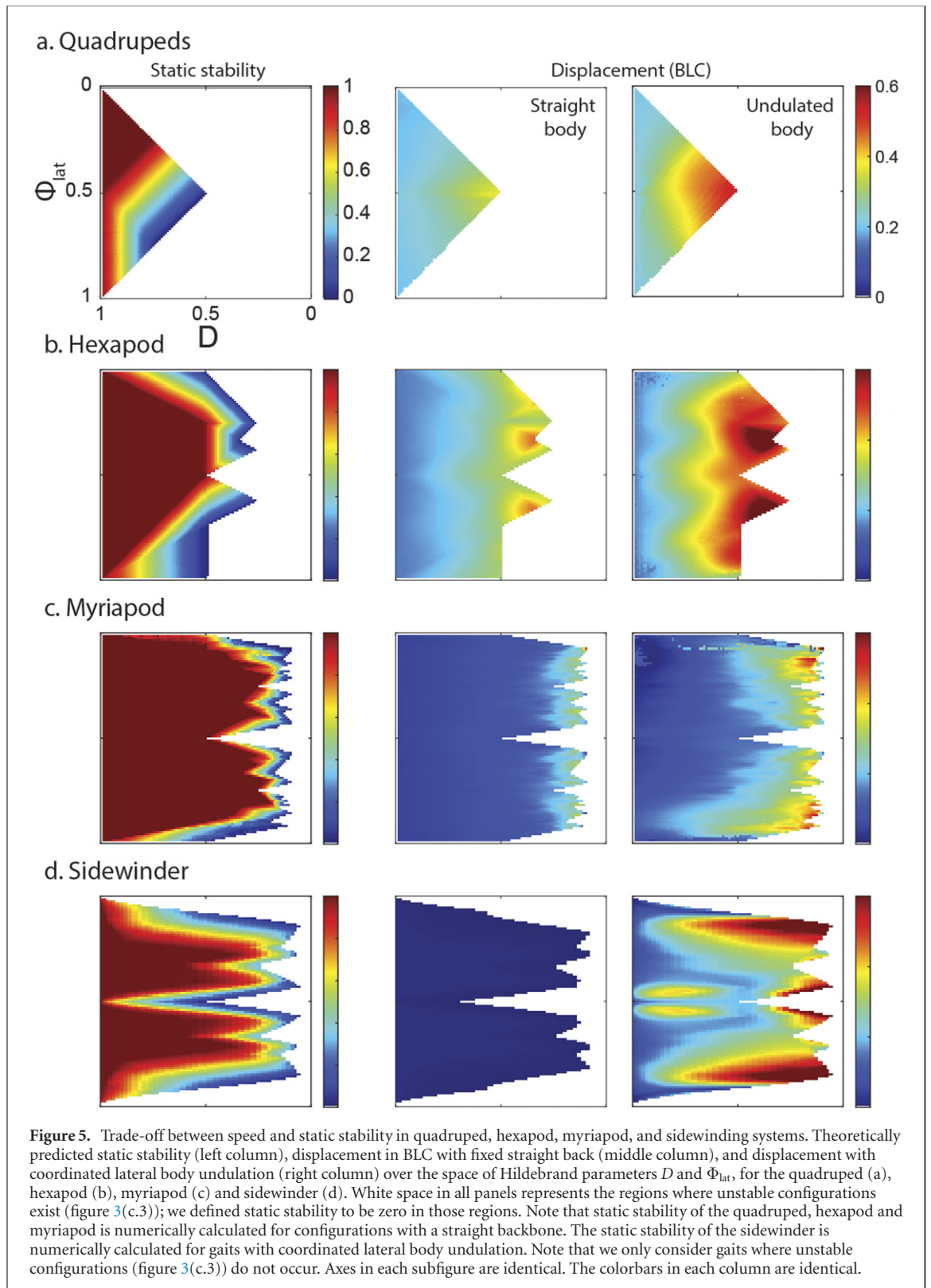
¹⁵ In the case of limbless sidewinding, unstable configurations are defined as those with no contact, see figure S4.

figure 5. As expected, when comparing the same gait parameters (duty factor D and phase lag Φ_{lat}) among different systems, the static stability increases with increasing number of legs. Similarly, an increase in duty factor results in an increase in static stability. Moreover, we observe that the DS ($\Phi_{lat} > 0.5$) is in general less stable than the LS ($\Phi_{lat} < 0.5$). Thus, most DS gaits are stable only for systems with many legs, such as myriapods.

Surprisingly, modulating the lateral phase lag only affects the static stability, while body speed is not correlated with the lateral phase lag. On the other hand, animals including myriapods [34] and quadrupedal lizards [20, 52, 53] have been observed to modulate the lateral phase lag as speed increases. In other words, in biological systems, the loss of static stability is compensated by a gain in speed while our findings indicate that speed is independent of lateral phase lag modulation. We hypothesize that this discrepancy is due to differences in body–limb coordination, which we consider in later sections.

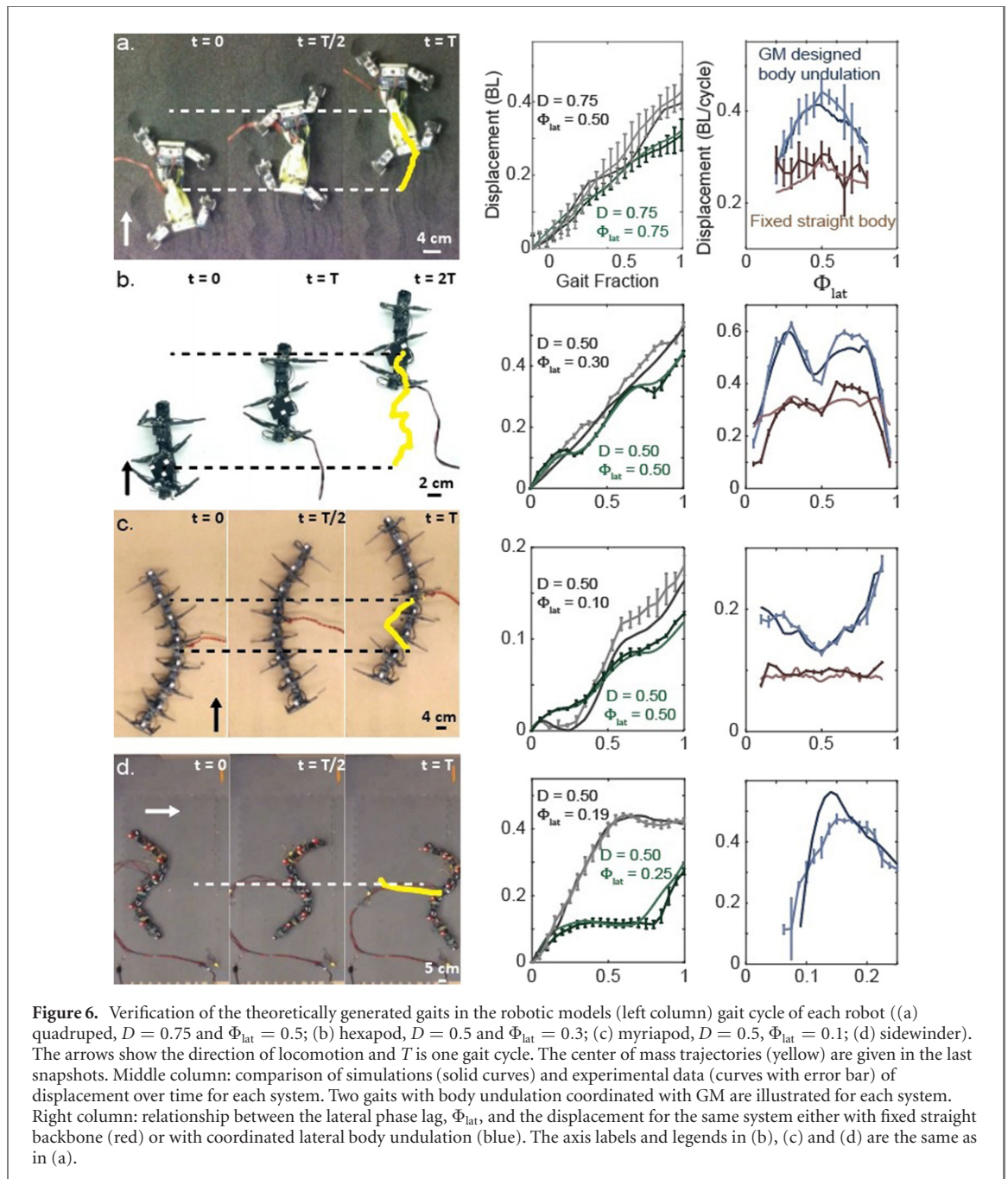
2.5. Testing speed and stability predictions in experiments

Using roboophysical models, we tested the locomotor performance of gaits with a range of lateral



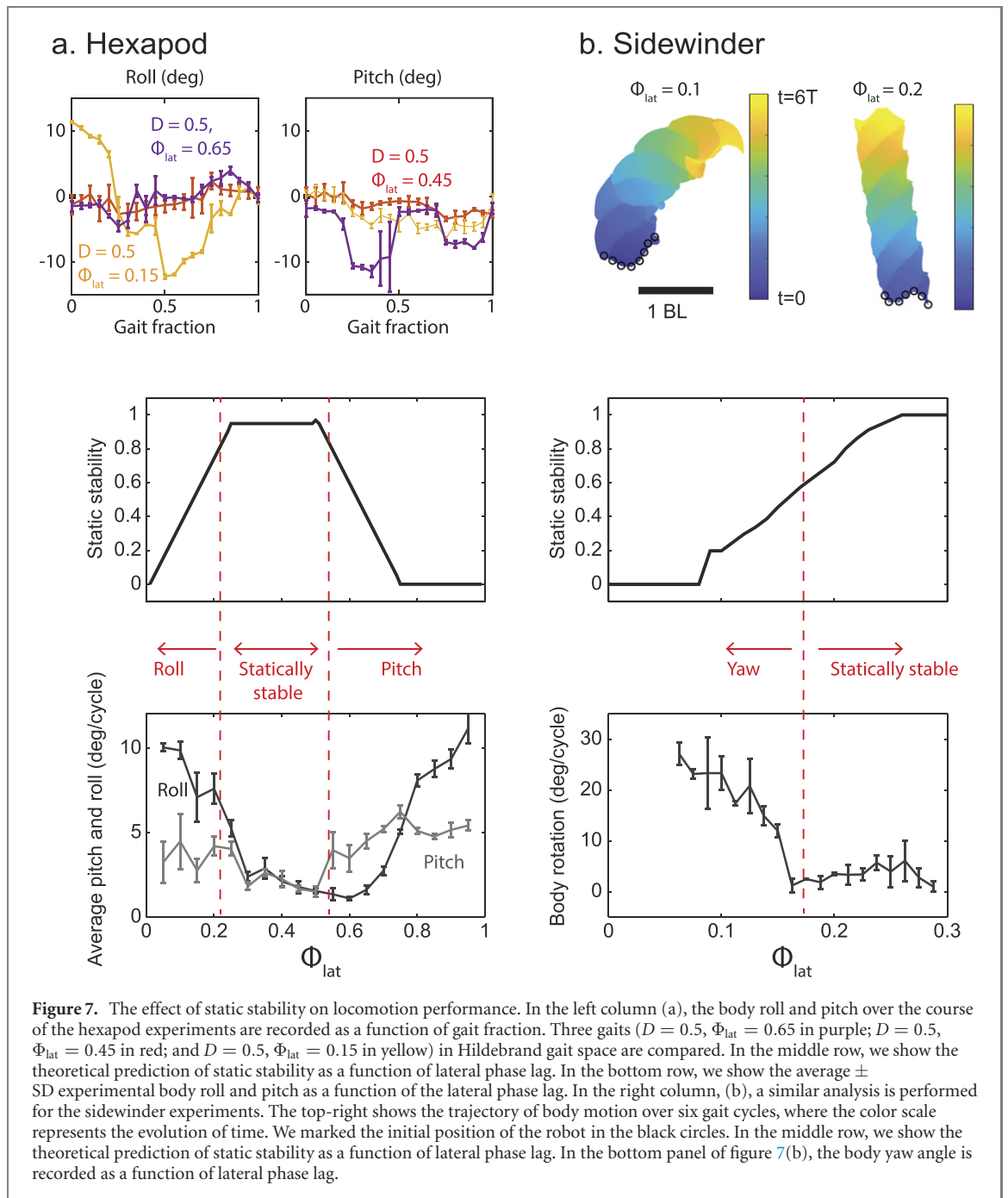
phase lags for quadruped, hexapod, myriapod, and sidewinder systems (figure 6). Hexapod, myriapod, and sidewinder experiments were performed on hard ground. Since quadrupedal systems in general are less statically stable, we posited that we would predominantly observe the effect of stability rather than the kinematics of gaits. Thus, for quadrupeds we tested performance on a granular medium (here, poppy

seeds) since the body is in contact with the substrate, ensuring static stability. The duty factor for the hexapod, myriapod and the sidewinder systems were fixed to $D = 0.5$, and the duty factor for the quadrupedal system was set to $D = 0.75$ for reference (see the materials and methods for additional experimental details). Note that in this section, there is no body undulation in any of the gaits.



We measured gait speed as the number of body lengths traveled per gait cycle. Interestingly, we observe that for statically stable gaits there is good agreement between the theory and experiments. Since our predictions are based on 2D calculations, they cannot capture 3D unstable behaviors, such as tipping over and falling to the ground. Therefore, we hypothesize that the discrepancy between our hexapod theory and experiments is caused by static instability. Note that our experiments on quadrupeds were performed on poppy seeds, where the ventral surface was often in contact with the environment. In our myriapod experiments, configurations tend to be mostly statically stable given their large number of legs. Therefore, the effect of static stability was only critical in our hexapod experiments.

To test our hypothesis that static stability is the source of the theory–experiment discrepancy, we characterized unstable behaviors by the roll and pitch of the robots. We recorded the body pitch and roll during the course of the robophysical hexapod gaits. The experimental data for these experiments over three gait settings ($D = 0.5\Phi_{\text{lat}} = 0.15$, $D = 0.5\Phi_{\text{lat}} = 0.45$, $D = 0.5\Phi_{\text{lat}} = 0.65$) are compared in figure 7(a). We observed that only the statically stable hexapod gait ($\Phi_{\text{lat}} = 0.45$) has both low pitch and low roll. The unstable hexapod gaits have either high roll angle ($\Phi_{\text{lat}} = 0.15$) or high pitch angle ($\Phi_{\text{lat}} = 0.65$). We calculated the average pitch and roll for each gait, and compared them with the numerical predictions of static stability. We observe that the range of low average pitch and roll overlaps with the



range of statically stable gaits. When the hexapod body is in configurations with low roll and low pitch, the experimental data agrees with the theoretical predictions.

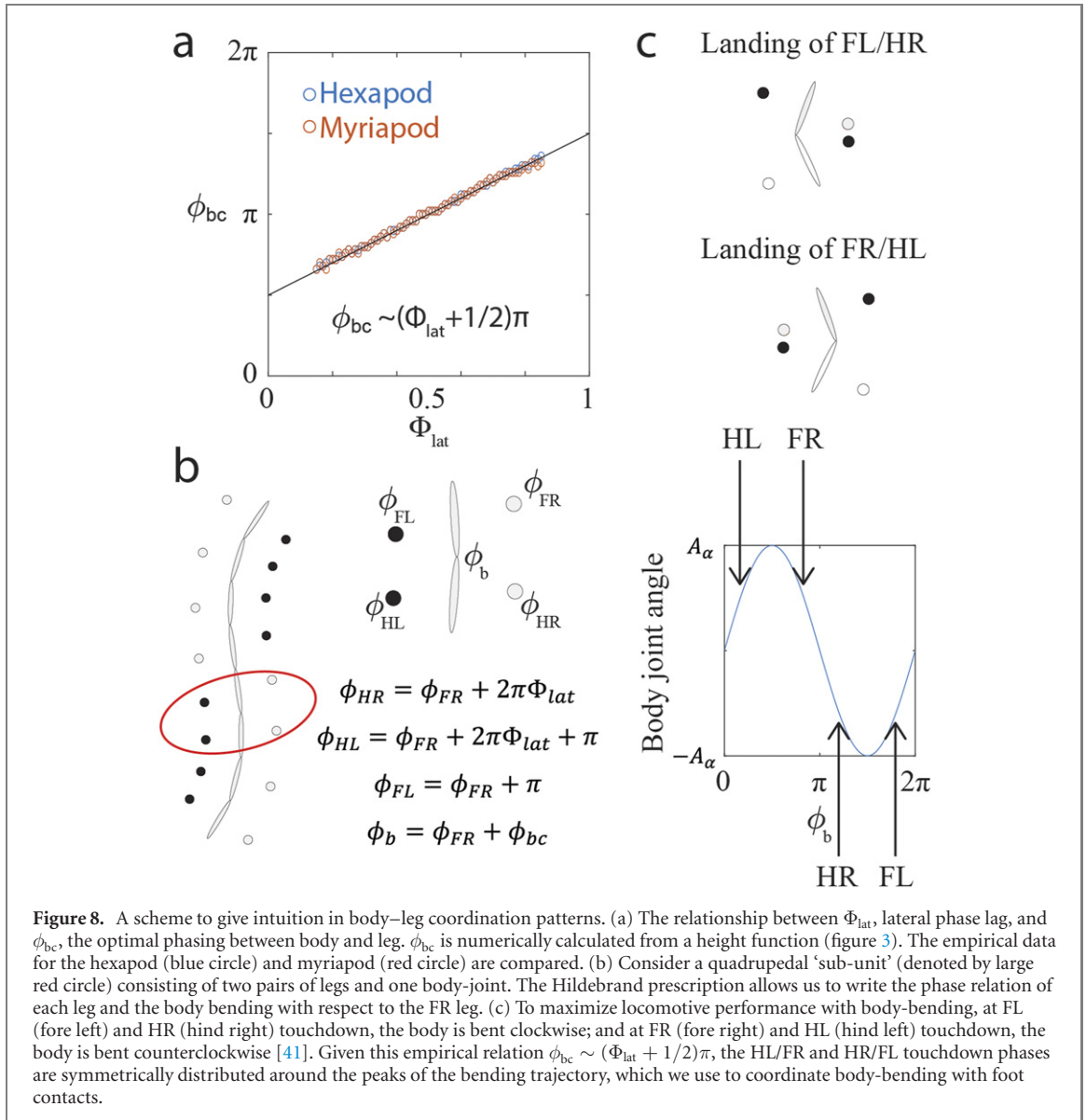
3. Body–leg coordination in Hildebrand gait formulation

3.1. GM to coordinate lateral body undulation

As discussed in the previous sections, speed is not correlated with the lateral phase lag when there is no body undulation. However, previous experimental gait studies with lizards and myriapods [20, 34] have found that modulation of lateral phase lag is associated with changes in lateral body undulation. For

example, lizards increase the amplitude of their lateral body undulation during transitions from LS walking to trotting or even DS gaits [20, 24, 25, 50, 52, 53]. Similarly, myriapods change their leg wave pattern (lateral phase lag) at high speed while simultaneously increasing lateral body undulation amplitude [34]. Accordingly, we hypothesize that modulating the lateral phase lag can regulate the balance between speed and stability if properly coordinated with lateral body undulation.

To account for these observations, we introduce the lateral body undulation by propagating a wave along the backbone from head to tail [54]. Note that we only consider the format of traveling wave body undulation here; relative advantages of standing



and traveling waves are discussed in [55]; and for quadrupeds, the number of body undulation DoF drops to 1, where the format of traveling wave body undulation essentially reduces to a standing wave. The body undulation wave is

$$\alpha(\phi_b, i) = A_\alpha \cos(\phi_b - 2\pi(i-1)\Phi_{lat}^b), \quad (4)$$

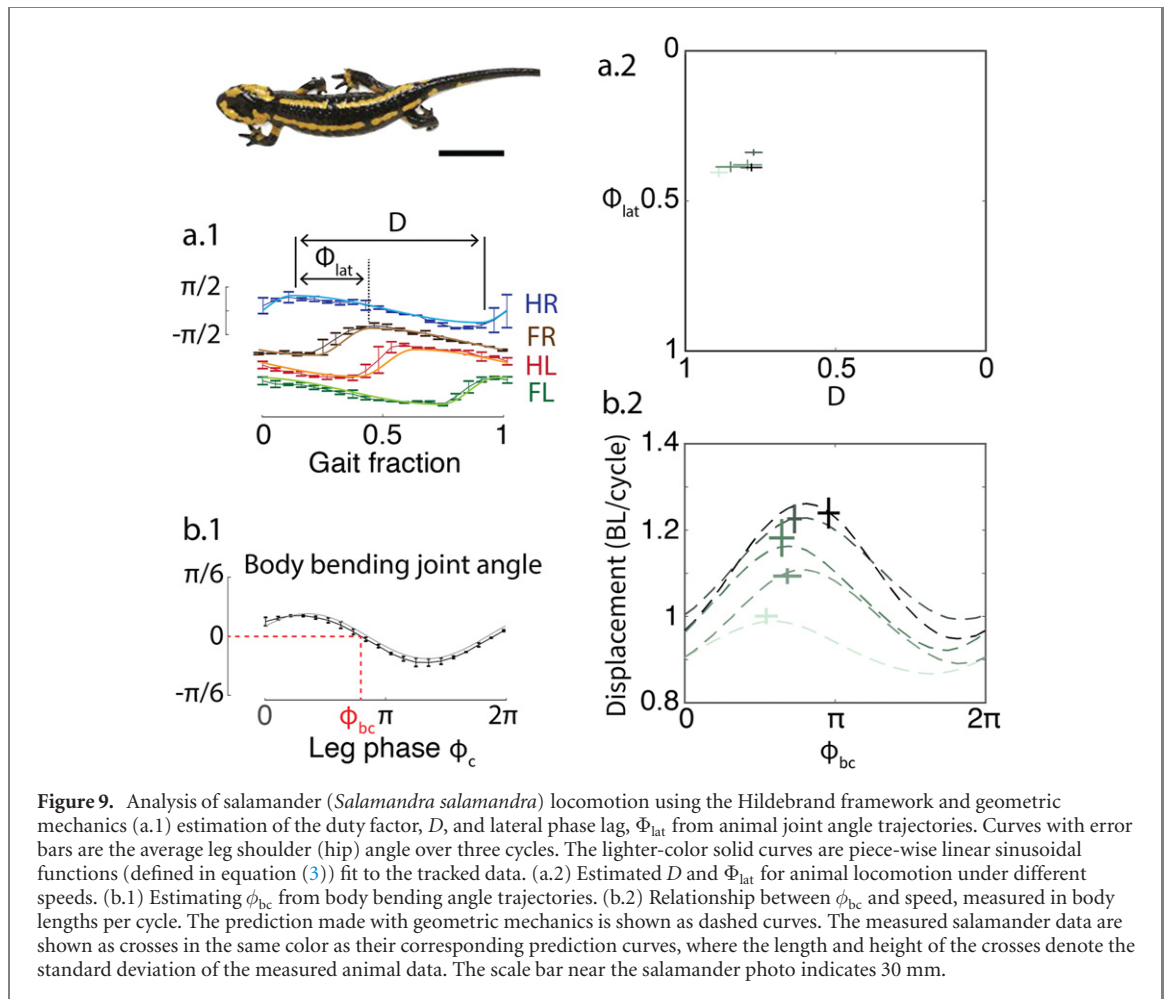
where $\alpha(\phi_b, i)$ is the angle of i th body joint at phase ϕ_b , $2\pi\Phi_{lat}^b$ is the phase lag between consecutive joints. For simplicity, we assume that the spatial frequency of the body undulation wave and the contact pattern wave are the same¹⁶, i.e. $\Phi_{lat}^b = \Phi_{lat}$. In this way, gaits of multi-legged locomotors by superposition of a body wave and a leg wave can be described as the phase of contact, ϕ_c , and the phase of lateral body undulation ϕ_b . These two independent phase

¹⁶ In most biological and robotic systems, the body undulation and the contact waves in general share the same spatial frequency. In some cases, such as sinus lifting in snakes [56], the body undulation and the contact waves differ.

variables represent a reduced shape space (see materials and methods) on a two-dimensional torus on which we can apply GM gait design techniques to optimize body–limb coordination.

The GM gait design framework [39, 42, 57] separates the configuration space of a system into two spaces: the position space and the shape space. The position space represents the location (position and orientation) of a system relative to the world frame, while the shape space represents the internal shape (joint angles) of the system. The GM framework then establishes a functional relationship to map velocities in the shape space into velocities in the position space; this functional relationship is often called a *local connection*. The curl of the local connection, which we call a ‘height function’, can then be used to design, analyze, and optimize gaits.

Using GM tools, we derived height functions and designed gaits (materials and methods). Figures 3 and 4 show examples of coordination between the lateral body undulation and contact



phase derived with GM. We also provided an example of coordinating the body undulation and contact pattern for sidewinding in figure S4 (<https://stacks.iop.org/BB/17/046015/mmedia>).

Once we design a coordination pattern $\phi_c \rightarrow \phi_b$ in the reduced shape space, we can convert that pattern into both a contact pattern and body undulation.

We quantified the body–leg coordination by its phase lag: $\phi_{\text{bc}} : \phi_c - \phi_b$. Interestingly, we observed that the empirically calculated ϕ_{bc} has a linear relationship with Φ_{lat} (figure 8). We next seek the physical intuition behind this relationship. We first decomposed the body–leg coordination to a single ‘subunit’, which we define as two pairs of legs and one body joint. Our Hildebrand-based approach then allows us to prescribe the phase of each feet and the body bending. Previous work [41] found that at the optimal body–leg coordination, the body is bent clockwise when the HL/FR feet land (respectively, counterclockwise for HR/FL). Deviation from this coordination can lead to a loss in speed. We can encode this relation by $\phi_{\text{bc}} \sim (\Phi_{\text{lat}} + 1/2)\pi$. Then, the FR and HL foot touch-down are symmetrically distributed around the peak of the clockwise body bending angle, and the touch-down of FL and HR feet are symmetrically distributed around the peak of

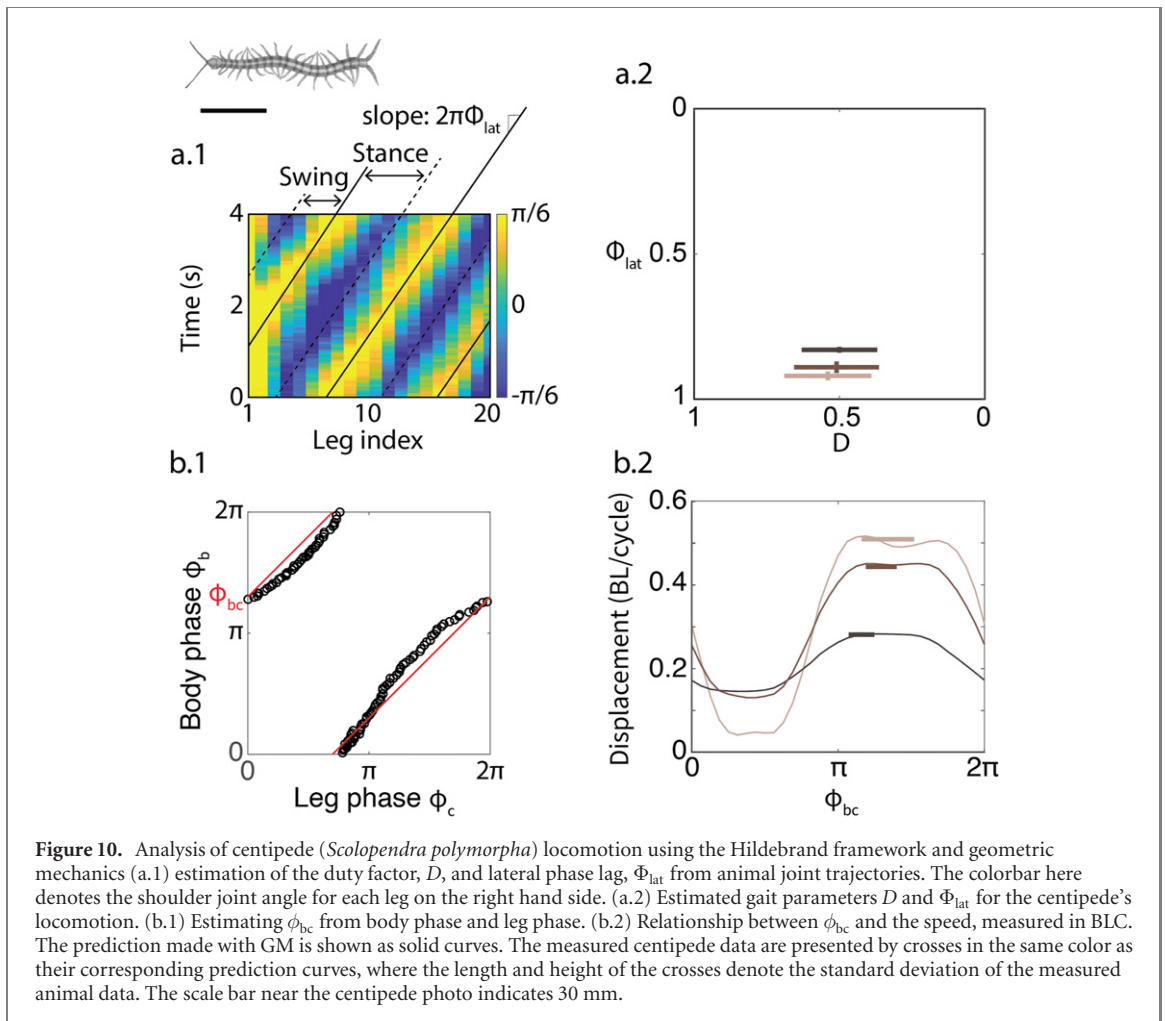
counterclockwise body bending angle. Via this relationship, we posit that despite the seemingly complicated whole-body motion, the optimal body–leg coordination is achieved by locally coordinating each sub-unit of two legs and a body joint.

3.2. Numerical prediction of speed and stability

We used numerical simulation to predict the gait speed and stability at a range of lateral phase lags and duty factors for the quadrupedal, hexapod, myriapod and sidewinder systems. We observed that modulating the lateral phase lag can regulate the balance between speed and stability if properly coordinated with lateral body undulation. The loss of static stability is compensated by a gain in speed only when the body and limb phases are properly coordinated. These observations were derived by plotting gait speed and stability against the extended Hildebrand gait parameters, shown in figure 5. The addition of body undulation slightly changes the static stability, as depicted in figure S1.

3.3. Testing predictions for body–leg coordination in experiments

We tested the locomotion performance of systems with discrete contact and coordinated lateral body



undulation using robophysical models (see materials and methods for details). We recorded the displacement over time for two gaits in each system (figure 6). Our numerical predictions quantitatively agree with experiments not only in the average displacement per gait cycle, but also in the time evolution of the displacement.

The only notable theory–experiment discrepancies occur in the hexapod and the sidewinder systems. As discussed earlier, static instability can lead to theory–experiment discrepancy for hexapods and sidewinders due to the planar assumptions made in our theoretical model. To investigate this discrepancy further, we studied the effect of static instability on sidewinders and observed that some gaits result in significant yaw (figure 7(b)), such that the robot's path deviates from the desired straight-line course. Comparing the net yaw change per gait cycle with the numerical predictions of static stability reveals that significant yaw only occurred in gaits with low static stability. As static stability increases (for sidewinding, stability increases with the lateral phase lag), the unmodelled turning vanished.

4. Body–leg coordination in biological locomotors

Symmetric gaits in quadrupedal animals can be categorized using Hildebrand analysis [22, 49]. Recent work revealed that a GM framework predicted optimal body–leg coordination for fire salamanders (*Salamanca salamandra*) [41, 58]. However, the means by which salamanders modify their leg movements and body–leg coordination in response to speed changes was previously unstudied. In this work, we recorded fire salamanders moving on sand. Five individuals were recorded, and their foot placement and backbone positions tracked. For simplicity, we only consider the body movement between the shoulder and the hip, for which salamanders exhibit a standing wave [59]. From the tracking data we measured gait parameters such as duty factor, lateral phase lag, amplitude of body bending, and amplitude of leg movements. We then used GM to predict the optimal body–leg coordination for salamanders walking at various speeds. We observed quantitative agreement between the GM prediction and the biological measurements (figure 9).

Beyond quadrupedal animals, our methods can also be applied to study animals with various numbers of legs and backbone segments. Centipedes are known to be fast-moving locomotors: certain centipedes are the fastest-running terrestrial arthropods [34, 51]. Given their high speeds, past work often used dynamic models to analyze their locomotion [60, 61]. We hypothesized that despite their high speeds, centipede locomotion could be analyzed with our quasi-static geometric model because of the high damping from many leg contacts. To test this hypothesis, we recorded videos of centipedes (*Scolopendra polymorpha*) moving at different speeds. Three individuals were recorded, their leg and body positions tracked, and their gait parameters estimated. We then used GM to predict the optimal body–leg coordination. We once again observed quantitative agreement between GM predictions and the biological measurements (figure 10).

5. Discussion and conclusion

5.1. Principles of gait modulation

In this paper, we developed a general gait design framework for a broad class of locomotors: multi-legged robots (with an arbitrary number of pairs of legs) with an articulated backbone, including limbless sidewinding. Specifically, we extended the Hildebrand gait formulation [22, 49], originally used to categorize symmetric quadrupedal gaits, and combined it with modern GM tools to investigate optimal leg–body coordination. We showed that the symmetry in Hildebrand quadrupedal gaits is conserved for other locomotors. The framework is not only simple enough to enable physical interpretation of the gait parameters, but also covers a range of potentially interesting gaits, offering a scheme to modulate gaits in a diverse range of robot shapes. These properties enable our framework to link well-studied locomoting systems like quadrupeds and hexapods with less-studied systems like myriapods, generating new opportunities to transfer insights among, and compare between, different locomoting morphologies. Given a new robot with arbitrary pairs of legs or without legs, our framework can immediately provide effective open-loop gaits, which can serve as the basis for closed-loop adaptive or data-driven/learning-based control algorithms.

Our gait identification and dimensionality reduction principles reveal insights into proper contact scheduling. These principles could serve as a starting point for additional layers within in a robot's control architecture or even for mechanical design iterations. [62] found that while direct application of gait design tools can prove ineffective in rough terrain, adding passive leg compliance can greatly improve performance in this environment. Our proposed framework can not only simplify the gait design and modulation

process for robots with different morphologies in various homogeneous environments, but can also be used to test hypotheses and therefore give novel insights into the control principles behind gaits in biological systems.

Finally, our framework facilitated testing hypotheses about the role of body undulation in multi-legged systems. These observations can act as guidelines in the control of a variety of legged robots. For example, in RHex [3], a hexapod with flexible legs attached to a rigid body, the duty factor (e.g., the relationship between swing and stance phase dynamics) is the primary tuning parameter that can regulate the balance between speed and stability. In other cases, such as in [63], a segmented robot with a flexible backbone and contralateral legs mechanically coupled to have opposite contact states (and therefore, to have a fixed duty factor $D = 0.5$), the lateral phase lag acts as the salient parameter to balance between speed and stability when properly coordinated with lateral body undulation. Additionally, body undulation also plays an important role in turning motions. Although not explicitly studied in this work, our framework can also be used to investigate the coordination between body movement and contact patterns during turning motions in both legged [41] and limbless [46] systems.

5.2. Insights from robotics to biological systems

We also demonstrated that once two gait parameters (duty factor and lateral phase lag) are specified, the gait can be prescribed and then analyzed with geometric tools. To explore gait tuning principles for locomoting systems, we quantitatively investigated the effect of modulating gait parameters on locomotor performance. As shown in figure 6, we found that in robots with a fixed straight backbone, the displacement per gait cycle is nearly invariant to the changes in the lateral phase lag, Φ_{lat} . On the other hand, in gaits where body undulation is properly coordinated with leg motions, Φ_{lat} affects the displacement. This seemingly counter-intuitive observation can help us develop hypotheses about gait modulation principles.

In addition to these robotics applications, our proposed control principles can also offer explanatory power to some hypotheses about biological locomotion. For example, biological myriapods (*Chilopoda*) can be categorized into direct-wave myriapods [34] and retrograde-wave myriapods [34]. Direct-wave myriapods propagate their leg contact wave from tail to head (corresponding to $\Phi_{\text{lat}} < 0.5$ in our modified Hildebrand formulation) while retrograde-wave myriapods propagate their wave from head to tail ($\Phi_{\text{lat}} > 0.5$) [48]. Interestingly, Manton [34] showed that there is no significant lateral body undulation in direct-wave myriapods regardless of their speed; instead, the only significant gait modulation at high

Table 1. The Ground Reaction Force (GRF) formulas used to model quadrupeds, hexapods, myriapods, and sidewinders robophysical systems.

Robot	GRF formula	References
Quadruped	Poppy seed RFT	[41, 64]
Hexapod	Anisotropic Coulomb friction	[65, 66]
Myriapod	Anisotropic Coulomb friction	[65, 66]
Sidewinder	Isotropic Coulomb friction	[43, 67]

speed is a decrease in duty factor. On the other hand, gait modulation in retrograde-wave myriapods is much more complicated: they not only decrease the duty factor, but also increase the lateral phase lag. More importantly, they exhibit characteristic lateral body undulation at high speeds [34, 51]. This observation is consistent with the principles discovered via our gait analysis methods, where we found that tuning the lateral phase lag can only improve the speed if accompanied with properly coordinated lateral body undulation (table 1).

6. Materials and methods

6.1. Geometric mechanics

We used tools from geometric mechanics, the application of differential geometry concepts to rigid body mechanics, to design the coordination between body undulation and contact patterns. In this section, we provide a concise overview of the tools used to design the coordination patterns. For a more detailed and comprehensive review, we refer readers to [41, 42, 57].

The GM gait design framework separates the configuration space of a system into two spaces: the position space and the shape space. The position space represents the location (position and orientation) of a system relative to the world frame, while the shape space represents the internal shape (joint angles) of the system. The GM framework then establishes a functional relationship to map velocities in the shape space into velocities in the position space; this functional relationship is often called a *local connection*.

6.1.1. Reduced equation of motion

In kinematic systems where frictional forces dominate inertial forces, the equations of motion can be approximated by:

$$\xi = \mathbf{A}(\Phi)\dot{\Phi}, \quad (5)$$

where $\xi = [\xi_x \ \xi_y \ \xi_\theta]^T \in g$ denotes the body velocity in the forward (x), lateral (y), and yaw (θ) directions; Φ denotes the internal shape variables. In this work, $\Phi = [\phi_c \ \phi_b]^T$, representing the contact phase and the lateral body undulation phase. $\mathbf{A}(\Phi)$ is the local connection matrix, which encodes environmental substrate interactions.

6.1.2. Numerical derivation of the local connection matrix

The local connection matrix \mathbf{A} can be numerically derived via force and torque balances [18]. The force and torque balance equations require a model of the ground reaction forces (GRF), such as granular material interaction and ground friction. We summarize the GRF formula for our four robots in Table 1. Further details on the local connection derivation can be found in the supplementary material.

6.1.3. Connection vector fields and height functions

Once we obtain the local connection matrix, we can further analyze the system kinematics during locomotion. Each row of the local connection matrix \mathbf{A} corresponds to a component direction of the body velocity. Each row of the local connection matrix over the shape space then forms a connection vector field. Then, the body velocity can be computed via the dot product of connection vector fields and the shape velocity $\dot{\Phi}$.

A periodic gait can be represented as a closed curve in the shape space. The displacement resulting from a gait, $\partial\chi$, can be approximated by:

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta \theta \end{pmatrix} = \int_{\partial\chi} \mathbf{A}(\Phi) d\Phi. \quad (6)$$

According to Stokes' theorem, the line integral along a closed curve $\partial\chi$ is equal to the surface integral of the curl of $\mathbf{A}(\Phi)$ over the surface enclosed by $\partial\chi$:

$$\int_{\partial\chi} \mathbf{A}(\Phi) d\Phi = \iint_{\chi} \nabla \times \mathbf{A}(\Phi) d\phi_c d\phi_b, \quad (7)$$

where χ denotes the surface enclosed by $\partial\chi$. The curl of the connection vector field, $\nabla \times \mathbf{A}(\Phi)$, is referred to as the *height function*. The three rows of the vector field $\mathbf{A}(\Phi)$ can thus produce three height functions in the forward, lateral and rotational directions, respectively.

The height function derivation simplifies the gait design problem to drawing a closed path in a Euclidean shape space. The body displacement from a path can be approximated by the integral of the surface enclosed by that path.

6.1.4. Toroidal shape spaces

In our gait prescription, the two shape variables are parameterized as cyclic phases, resulting in a toroidal shape space (T^2) [68]. Examples of height functions on toroidal shape spaces are shown in figure 3(b). The shape variables $\Phi = [\phi_c, \phi_b]^T \in T^2$ correspond to the phase of contact and the phase of the lateral body undulation, respectively. A gait is a closed curve in the toroidal shape space (solid purple curve figure 3(b)), but as it is a non-Euclidean space, there is no clear 'surface' enclosed by the path.

To form an enclosed surface, Gong *et al* [42] introduced the notion of ‘assistive lines’ in the Euclidean parameterization of the toroidal shape space. This method allows a surface integral to be calculated. In figure 3(b), the surface integral is the area within solid lines subtracted from the area of the surface enclosed in the upper left corner. For simplicity, in our optimization we assumed that the mapping between the two phase variables is linear and that the body and legs share the same temporal frequency, i.e., $\phi_b = \partial\chi(\phi_c) = \phi_c + \phi_0$, where ϕ_0 is the phase offset between lateral body undulation and contact pattern to be optimized.

6.2. Simulation

We performed a numerical simulation to predict locomotive performance, and compared these results to those obtained from robophysical experiments. Specifically, we prescribed the contact state and the joint angle of each leg by a single variable, ϕ_c , using equations (2) and (3). Similarly, we prescribed the lateral body undulation by another variable, ϕ_b , using equation (4). The amplitudes of leg and body joint angles are listed below.

Robot	A_θ	A_α
Quadruped	30°	30°
Hexapod	10°	10°
Myriapod	12°	17°
Sidewinder	N/A	5.6L (rad)

Note that the amplitude of sidewinder body undulation is related to the lateral phase lag, such that the ‘relative curvature’, that is, the maximum curvature of the backbone of limbless locomotors [43, 44, 69], remains constant.

Given the body–limb coordination function $\partial\chi: \phi_c \rightarrow \phi_b$ as described above, the shape variable Φ and shape velocity $\dot{\Phi}$ can be rewritten as:

$$\Phi = \begin{bmatrix} \phi_c \\ \partial\chi(\phi_c) \end{bmatrix}, \quad \dot{\Phi} = \begin{bmatrix} 1 \\ \frac{d\partial\chi(\phi_c)}{d\phi_c} \end{bmatrix} \dot{\phi}_c. \quad (8)$$

The body displacement is computed by integrating the following ordinary differential equation [39]:

$$g(t) = \int_0^t T_e L_{g(\phi_c)} A(\Phi) d\Phi \quad (9)$$

$$= \int_0^t T_e L_{g(\phi_c)} A \left(\begin{bmatrix} \phi_c \\ \partial\chi(\phi_c) \end{bmatrix} \right) \begin{bmatrix} 1 \\ \frac{d\partial\chi(\phi_c)}{d\phi_c} \end{bmatrix} d\phi_c, \quad (10)$$

where $g = (x, y, \alpha) \in \text{SE}(2)$ represents the body frame position and rotation [70]. Note that $T_e L_g$ is the left lifted action with respect to the coordinates

of g :

$$T_e L_g = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

Integrating the ordinary differential equation throughout one period (from $\phi_c = 0$ to $\phi_c = 2\pi$) results in the body trajectory, from which we can determine the predicted displacements in the forward, lateral, and rotational directions over one gait cycle. Note that we neglect any inertial effects in this simulation.

6.3. Robophysical experiments

6.3.1. Robotic models

All of the robophysical models were designed in Solidworks and printed using Stratasys dimension Elite 3D printer. They are powered with an external power supply (12 V, 5 A) and controlled via the MATLAB DYNAMIXEL SDK, interfacing with the servo motors through a Robotis USB2Dynamixel controller. All the robots have open-loop control such that gait parameters are not changed during an experiment, and the control signals (servo positions) continue to be sent as a function of time, regardless of external forces or the tracking accuracy of the servos.

The quadruped robot (figure S4(a), 450 g, ~ 40 cm long [71]) has four legs and an actuated trunk. Each limb is actuated with two Dynamixel XL-320 servos (stall torque 0.39 (N m)) to control the vertical position and the step size of the leg (45 mm-high steps, figure S4(a)). The body joint servo (Dynamixel AX-12) controls the horizontal bending. The legs have a cube shape with 24×24 mm² surface area.

The hexapod robot (figure S4(b), 300 g, ~ 25 cm long) has a segmented body (three segments) with pairs of legs in each segment. The vertical and horizontal motion of the legs in a segment are coupled (out-of-phase) and controlled by two Dynamixel XL-320 servos (figure S4(b)). The body joint servos (Dynamixel XL-320) controls the horizontal bending of the segments. The legs have pointed feet.

The myriapod robot (figure S4(c), ~ 1000 g, ~ 72 cm long) has a eight body segments, similar to the segments of the hexapod robot [72]. Each segment has a pair of rigidly connected legs 12 cm in length. There are three servos (Dynamixel XL-320) in each segment; one controls horizontal body bending and two control the fore/aft and up/down motion of the legs (figure S4(b)).

The sidewinder robot has seven segments, each of which contains two joints connected at an angle of 90° (figure S4(c)). Each joint is comprised of a AX-12 servo motors (stall torque = 1.5 (N m)). The horizontal motors vary the lateral wave, and the vertical motors create a changing contact pattern.

6.3.2. Experimental setups and data analysis

We used an Optitrack motion capture system (including 4–6 Naturalpoint, Flex13 cameras, 120 fps and

Motive software) to capture the position and orientation of the reflective markers attached to the robots. The data was analyzed in Matlab.

Quadruped robot experiments were performed on a trackway filled with ~ 1 mm diameter poppy seeds [71]. Before each experiment we fluidized the bed using four vacuums to prepare a uniform loosely packed state. Each experimental condition, consisting of the quadruped robot and a set of gait parameters was repeated three times for a total of nine gait cycles.

The hexapod and myriapod robot experiments were performed on a cardboard and particle board surface, respectively. Before each experiment, the joints were set to their neutral positions. The robots were allowed to run for three cycles (five trials/gait).

Sidewinder experiments were performed on a foam mat surface to reduce slip. Each experiment was started from the same position and repeated three times, for five to six gait cycles per trial.

6.3.3. Salamander data analysis

In the salamander experiments, individual animals walked along a straight trackway filled with 300 μm glass particles. Three cameras (GoPro Hero3+, 720 pixel resolution) were positioned around the trackway and recorded synchronized videos at 120 FPS. All experiments were approved by the Royal Veterinary College's Clinical Research Ethical Review Board, approval number 2015 1336. Salamanders were captured under collection permit # 2016/001092 provided by the Government of the Principality of Asturias. No animals were harmed for the experiments, and animals had rest periods in between data collection trials. Experiments were conducted in a humidity-controlled laboratory at the University of Oviedo, Spain. The temperature (18 °C) and light cycle (12 h dark, 12 h light) were maintained at constant levels.

At least three gait periods were recorded in each experiment. Limb positions, body angles, and footfall timing are manually extracted from each recording. We fitted the animal body angles with the first two terms of Fourier series as in [41].

6.3.4. Centipede data analysis

In the centipede experiments, individual animals ran along a flat, hard trackway. One overhead camera recorded centipede locomotion. Positional data were extracted from videos with animal pose estimation software DeepLabCut (DLC) [73]. Ten frames from each video were extracted and manually labeled. DLC would then provide positions for labeled points on all of the other frames. The positions of feet and body segments were labeled.

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Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: <https://arxiv.org/submit/4145491/view>.

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